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Numerical analysis of low-height Sodar echoes

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1. *Introduction.*

The Sodar system that provides the data considered in this report is placed in Turin and is jointly operated by the Istituto di Elettronica of the Politecnico and the Istituto Elettrotecnico Nazionale « G. Ferraris ».

The system is composed of an acoustic antenna, a 1.2 m paraboloid, of a 100 W (electric) transmitter of 50 msec pulses at 2000 Hz, and of a low noise receiver. The parabolic antenna is placed on the top of the building at 25 m from street level and is pointing vertically: it operates both as a transmitter and a receiver. The 50 msec pulses have a repetition period of 5 sec. The echo is received only after 200 msec from the instant of transmission in order not to saturate the receiver. The envelope of the received signal, that is proportional to the acoustic pressure, is feeded to an A/D converter and is sampled at a frequency of 400 Hz: the result is the digitized echo envelope (PERONA, PISANI & MAZZETTI, 1975; PERONA & PISANI, 1976). This quantity, a, is averaged over 10 successive soundings and then plotted on a logarithmic scale, as represented in Fig. 1, where $\mathcal{L}$ is equal to:

$$\mathcal{L} = 10 \log_{10} \left[\frac{\sum_{1}^{10} a_i}{10} \right] \tag{1}$$

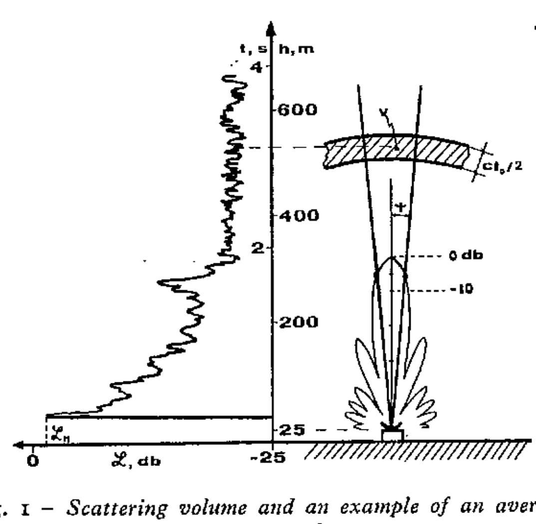

Fig. 1 – *Scattering volume and an example of an averaged received echo.*

In Fig. 1 the time delay or, equivalently, the scattering height of the signal is represented along the vertical axis.

In order to obtain information with regard to the atmospheric turbulence of the lower levels, a set of quantities that characterize the trend of each echo needs to be found. These quantities could be: the exponent of the power law approximating the returned signal as a function of height; the height interval of validity of each approximating curve; the maximum level, in dB, of the plotted echo, that is, the level at the « minimum height of detection »; the maximum deviation of the signal fluctuations from the approximating curve; etc.

So far, these quantities do not appear to have been previously recorded in a systematic way. In what follows one week of data have been analyzed in accordance with an empirical method descrived further on, in order to show the usefulness of the method itself and its possibilities to be conveniently performed with automatic procedures by a microcomputer.

2. *Propagation theory.*

The acoustic sounder transmits a pressure wave and receives an echo backscattered by the atmospheric turbulence.

Using the « Radar equation », the power received by the scattering process is:

$$P_s = P_0 \frac{G_0 A}{4\pi} \int_V \frac{1}{r^4} \sigma_0 \, g \, dV \tag{2}$$

where P_0 is the transmitted acoustic power; G_0 is the maximum gain of the transmitting paraboloid; A is the effective area of the transmitting paraboloid, that is: $A = G_0 \lambda^2/4\pi$; g is the normalized gain in a given direction; V is the scattering volume; σ_0 is the backscattering cross-section per unit scattering volume and per unit solid angle.

Since the sounder makes use of pulses of finite length l and the antenna has its own directivity, the atmospheric volume contributing to backscattering, at a fixed instant, is represented in Fig. 1; it has a radial size equal to $c\,t_0/2$ (where c is the velocity of sound propagation and t_0 is the pulse length).

Therefore the integral in eq. (2) may be easily evaluated assuming σ_0 constant in the scattering volume and g constant and equal to 1 in the main lobe of the antenna and zero otherwise:

$$P_s = P_0 \frac{G_0 A}{4\pi} \int_V \frac{1}{r^4} g \, \sigma_0 \, r^2 \, d\Omega \, dr \tag{3}$$

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$$\cong \frac{P_0\, G_0\, A}{2\, h^2}\, \sigma_0\,(h)\, \Psi\, \frac{c\, t_0}{2} \tag{3}$$

where h is defined in Fig. 1 and some obvious approximations have been performed; Ψ is the 3 dB aperture of the antenna main lobe.

In eq. (3) it is to be noted that the quantities P_0, G_0, A, Ψ, c, t_0 are characteristic constants of the system used, whereas σ_0 depends upon the atmosphere present in the sounded zone.

The eq. (3) is valid if the whole volume intercepted by the antenna main beam takes part in the scattering; however, if a small zone of such a volume, having transverse size constant with height, gives contribution to the scattering process, the equation for the received power assumes a different aspect:

$$P_s \cong P_0\, \frac{G_0\, A\, V_0}{4\, \pi}\, \frac{\sigma_0\,(h)}{h^4} \tag{4}$$

where

$$V_0 = \alpha_0\, \frac{c\, t_0}{2} \tag{5}$$

and α_0 is the transverse area of the volume containing the backscattering turbulence.

3. *Turbulence theory.*

In the backscattering conditions of the measurements the scattering cross-section σ_0 may be evaluated as follows (TATARSKII, 1971)

$$\sigma_0 = b\, k_0^{1/3}\, \frac{C^2_T}{T^2_0} \tag{6}$$

where b is a numerical constant, k_0 is the wavenumber of the incident acoustic field ($k_0 = 2\,\pi/\lambda_0$, where λ_0 is the wavelength), C_T is the structure constant of the turbulence, due to the temperature fluctuations, T_0 is the mean temperature of the atmosphere.

It is well known that, in the atmosphere, the mean temperature is a function of the altitude; furthermore its vertical gradient determines the value of C_T, as appears in the turbulence theory, developped by several authors (TATARSKII, 1971; LITTLE, 1969).

The received power P_s and, consequently, the envelope of the pressure field, a, depends upon the height where, at any instant, the transmitted pulse is present:

$$a\,(h) \propto \frac{\sqrt{\sigma_0\,(h)}}{h^n} \tag{7}$$

where $n = 1$ for extended scattering (eq. (3)) and $n = 2$ for reduced scattering (eq. (4)), that is when

It is to be noted that the acoustic sounder is situated on the top of a building 25 m high, and h should be measured from such a level; however, for simplicity h will be measured from ground level.

The structure function C_T can assume the following form:

$$C^2_T = C\, \frac{d\vartheta_0}{dh}\, \varepsilon^{-1/3} \tag{8}$$

where: C is the constant of proportionally, function of the heat flux and independent from height; ϑ_0 is the mean potential temperature of the atmosphere, function of h; ε is the energy dissipation rate, function of the heat flux and the gradient of the mean wind velocity, that in turn are functions of h.

In presence of instabilities caused by heat flux rising from the ground, we can assume ε constant with height h: consequently, C_T depends upon the temperature gradient only.

Just to give an example of a particular physical situation and of the way to be followed in order to analyze it, let us assume that convection is present in atmosphere: this situation appears when from the ground, or from the buildings, air warmer than the neighbouring rises, cooling itself in the process because it contributes to the microscopic agitation of the fluid.

In convective conditions the gradient of the mean potential temperature decreases as $h^{-4/3}$, with a proportionality coefficient dependent upon the heat flux emitted from the ground (LUMLEY-PANOFSKY, 1964; TATARSKII, 1971):

$$\frac{d\vartheta_0}{dh} \cong B\, h^{-4/3} \tag{9}$$

where B is the proportionality constant.

Therefore the coefficient C_T, expressed by (8), with $\varepsilon = \text{const}$, depends on h in the following way:

$$C_T \propto h^{-2/3}. \tag{10}$$

Using eq. (9), the mean temperature profile comes out, somewhat above the zero level:

$$T_0\,(h) \cong T_0\,(0) - 3\, B\, h^{-1/3} - \Gamma\, h \tag{11}$$

where $T_0\,(0)$ is the mean temperature at the ground and Γ is the adiabatic lapse rate.

Let us make use of the results (10) and (11) in (7) to establish the form of the received signal as a function of the height from which it is scattered.

We will identify, in (10) and (11), the altitude measured from the street level with the height h measured from the sounder level.

a deviation of only $5 \div 10$ °K with respect to the ground value of approximately 300 °K: therefore in (6) the term $T(h)$ will be substituted simply by $T_0(0)$.

For an extended scattering volume, the backscattered signal depends upon h through the — 5/3 power:

$$(12) \qquad a(h) \propto h^{\alpha}, \qquad \alpha = -5/3.$$

On the contrary, if the scattering volume is small in comparison to the volume intercepted by the main sounder beam, the dependence of the backscattered signal by h is the following:

$$(13) \qquad a(h) \propto h^{\alpha}, \qquad \alpha = -8/3.$$

Of course, this same height dependence may arise because of different vertical structure in the temperature profile. Let us not forget that eqs. (9), (10) and (12) have been derived starting from a particular physical situation, and have been presented just to give an example, for a well defined model, of the analytical procedure to be followed to link Sodar data and atmospheric theory.

4. *The temperature profiles.*

In order to identify the functions (12) and (13) in the signal registrations, we have drawn the set of curves of Fig. 2 on transparent paper and we have superposed it on the registrations: the results are reported in section 5.

In Fig. 2 the curves labelled $\alpha = -5/3$ and $-8/3$ correspond respectively to eqs. (13) and (12), whereas the curves $\alpha = 0$ and $\alpha = -1$ have been drawn for convenience.

However, it is to be pointed out that all curves presented may just approximate experimental results. Therefore at this point it seems more plausible to assume that different height dependence may be connected to different temperature structure rather than to a more or less limited extent of the turbulence with respect to the antenna beam. Therefore, assuming that the C_T dependence of h is due only to the temperature gradient (this assumption may be correct only for thermal convection (TATARSKII, 1971), whereas the other factors are constant, we find that:

$$(14) \qquad \frac{d\vartheta_0}{dh} \propto h^{2(1+\alpha)}, \qquad \alpha < 0.$$

Integrating the previous equation, the function $\vartheta_0(h)$ comes out to be:

$$(15) \qquad \vartheta_0(h) \propto \frac{h^{3+2\alpha}}{3+2\alpha}, \qquad \alpha < 0$$

where we have omitted the proportionality constant and the mean potential temperature value at $h = 0$.

According to the convection theory (DERR, 1972), we expect that near the ground, just above a forced convection layer, there is an atmospheric layer where the mean potential temperature decreases with a strong gradient, whereas higher up there is a layer with the mean potential temperature constant (this layer is generally capped by an inversion).

Therefore in the intermediate layer the function $\vartheta_0(h)$ can be written as

$$(16) \qquad \vartheta_0(h) \simeq \vartheta_0(0) - \Theta\, h^{3+2\alpha}$$

where $\vartheta_0(0)$ is the value of the mean potential temperature at the ground, and Θ is a constant of proportionality.

Figure 3 shows the behaviour of the potential temperature represented by eq. (16). Of course, this figure is just indicative. It has been assumed that the atmosphere becomes adiabatic at the height h_Γ, and from this level the potential temperature has been drawn, according to eq. (16) and for the values indicated for α, down to the minimum level of detection possible with our instrument. Of course, a real situation may be much more complex than any particular curve drawn in the figure. Finally it is always to be remembered that the acoustic echo is propor-

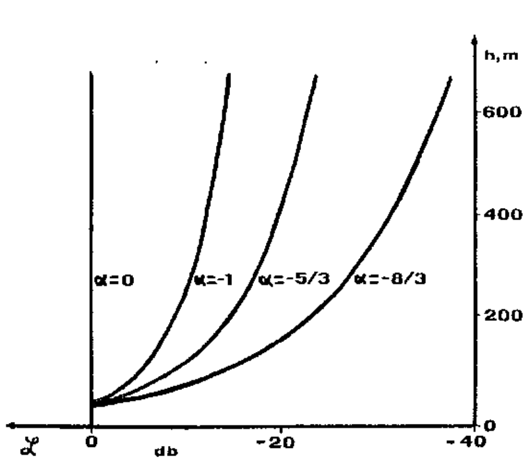

Fig. 2 – *Test curves of the α values indicated.*

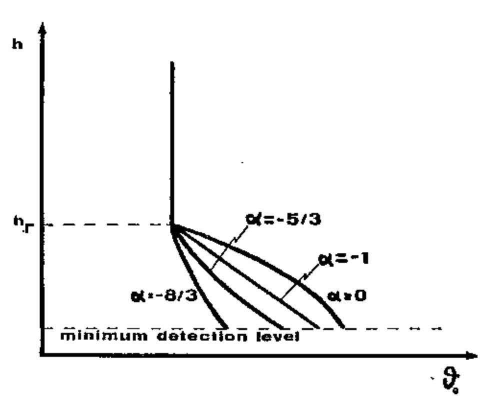

Fig. 3 – *Potential temperature v. height for different power laws.*

tional to the absolute value of the potential temperature gradient even though, for simplicity, in Fig. 3 only cases of superdiabatic atmosphere have been presented.

The mean actual temperature is dependent on the potential temperature:

$$T_0 = \vartheta_0 - \Gamma h \tag{17}$$

where Γ is the adiabatic lapse rate.

5. *Results.*

The system automatically performs one sounding per hour and plots the echo on paper in the way indicated in Fig. 1.

On each registration we have superposed the curves of Fig. 2 to find the best approximation and the height range of validity. Furthermore, the maximum value of the echo, that is the echo level at the maximum detection height, has been given: this quantity represents, in dB, the value of the proportionality constant in the received echo of eq. (12) and depends upon the vertical heat flux only, in the case of thermal convection.

The registrations of the days 1, 2, 3, 4, 7, 8, 9 of March 1977 have been examined: for the first four days 24 soundings per day were available, whereas for the next three days only the soundings during the middle hours of the day were available. The mean results relative to the considered time period are represented in the Figs. 4 ÷ 6. In the daily diagrams we have assembled the data corresponding to 2 successive hours in one single point.

The central diagram in Fig. 4, represents the probability $p(\alpha)$ of finding the curve α during the day. The quantity $p(\alpha)$ has been calculated as the ratio between the number $n(\alpha)$ of cases when the curve «α» has been found in the corresponding 2h interval and the total number N of soundings at the same hour, i.e.

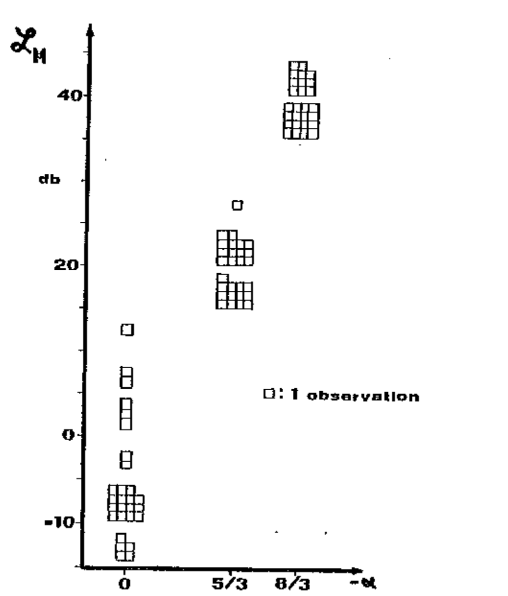

Fig. 5 – *Maximum level, $\mathscr{L}_M$, observed v. α.*

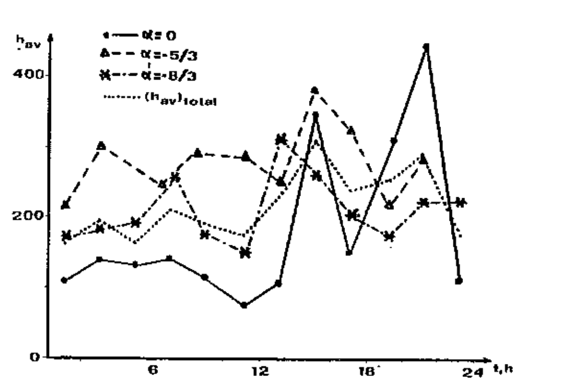

Fig. 6 – *Mean altitude of the layer approximated by a given «α» curve v. time.*

$$p(\alpha) = \frac{n(\alpha)}{N}. \tag{18}$$

The number N of soundings performed in a range of 2 hours, during all the days examined, has been drawn in the lower part of Fig. 4. It is interesting to observe that the maxima of the probability curves $p(\alpha)$ do succeed in time during the morning for increasing values of α (the maximum for $\alpha = 0$ is around 5 o'clock, for $\alpha = -5/3$ at 7 o'clock and for $\alpha = -8/3$ at 11 o'clock). Viceversa, the reserve order is found in the afternoon (for $\alpha = -8/3$ at 15 o'clock, for $\alpha = -5/3$ at 17 o'clock and for $\alpha = 0$ at 19 o'clock).

The curve α_{av} in the upper part of Fig. 4 represents the average value of the exponent α, at a certain hour during the day, that is:

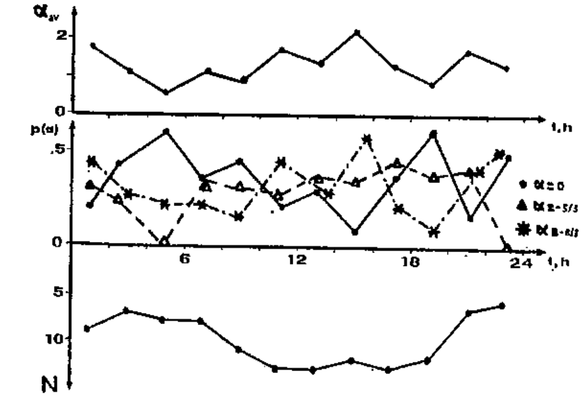

Fig. 4 – Probability p (α) f...